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Research efforts were directed towards a number of different topics connected with numerical methods for incompressible fluid flows. The subject groupings were as follows: 1. Finite element techniques (7 Papers) Stability of discretizations, Stream function methods and pressure recovery, Nonconforming schemes. 2. Vortex techniques (3 Papers) Higher order vortex algorithms, Analysis and computation. 3. Solution algorithms (5 Papers) Deflated conjugate gradients, Iterative methods for arbitrary meshes, Domain decomposition methods. 4. Fluid mechanics and phase transitions (3 Papers) Cahn-Hilliard equation analysis and algorithms, Stationary and evolutionary cases. 5. complementary volume methods (2 Papers) Vorticity-velocity methods, Primitive variable methods. 6. Control theory (1 Paper).

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EFFICIENT FINITE ELEMENT SOLUTION OF NAVIER-STOKES EQUATIONS AND RELATED TOPICS

Final Technical Report

for Grant No. AFOSR-84-0137

Principal Investigator: R. A. Nicolaides

AFOSR-TR- 89-1180

Research efforts were directed towards a number of different topics connected with numerical methods for incompressible fluid flows. The subject groupings were as follows:

1. Finite element techniques
   - Stability of discretizations
   - Stream function methods and pressure recovery
   - Nonconforming schemes
2. Vortex techniques
   - Higher order vortex algorithms
   - Analysis and computation
3. Solution algorithms
   - Deflated conjugate gradients
   - Iterative methods for arbitrary meshes
   - Domain decomposition methods
4. Fluid mechanics and phase transitions
   - Cahn-Hilliard equation analysis and algorithms
   - Stationary and evolutionary cases
5. Complementary volume methods
   - Vorticity-velocity methods
   - Primitive variable methods
6. Control theory

A brief description of these projects follows.

1.1. We showed that certain low order finite elements including the bilinear-constant pair are not convergent in general. Rigorous proofs and analysis were presented along with certain (not totally) satisfactory) remedies. This work is reported in:

Counterexample to uniform stability of bilinear/constant velocity pressure elements for viscous flows. (with J. Boland) Num. Math. 44 (1984)

Stable and semi-stable low order finite elements for viscous flows. (with J. Boland) SIAM Jnl. Num. An. 22 (1985)

1.2. The stream function formulation of the incompressible Navier-Stokes equations was analyzed. A new method for computing the pressure field from the computed velocity field was introduced and shown to be optimally accurate. These results are presented in:

Mixed finite element approximations for the biharmonic equation. (with G. Fix, M.D. Gunzburger, J. Peterson) Proc. 4th Intnl. Conf. Fin. Elem. Fluids, Univ. Texas Press (1984)

Finite element technique for optimal pressure recovery from stream function formulation of viscous flows. (with E.M. Cayco) Math. Comp. (1985)

Analysis of nonconforming stream function finite element methods for Navier-Stokes equations. (with E.M. Cayco) Comp. Math. and Applics. (to appear)

1.3. Two invited surveys on finite elements for viscous flows were prepared. They are:

Some aspects of finite element approximations of incompressible viscous flows. (with M.D. Gunzburger) Recent Advances in Num. Meth. in Fluids III, Pineridge Press (1984)

Algorithmic and theoretical results in computing incompressible viscous flows by finite element methods. (with M.D. Gunzburger) Comp. Fluids. 13 (1985)

2. Vortex methods offer an attractive alternative to finite difference and finite element methods for certain classes of flows. We proposed several new classes of more accurate vortex algorithms. One of the most promising of the new methods was rigorously analyzed. This method was implemented and the theoretical predictions were verified. These results were presented in:

Construction of higher order accurate particle and point vortex methods. App. Num. Math. 2 (1986)

Vortex multiple methods for incompressible viscous flows. (with S. Choudhury) Proc. 10th Intnl. Conf. Num. Meth. Fl. Dyn. (Beijing 1986)

Analysis of a higher order point vortex method for the incompressible Euler equations in two dimensions. (with Chichia Chiu) Math. Comp. (1988)

3. Solving the huge discrete equation systems resulting from discretization remains a major problem of computational mechanics. A new method for improving the convergence of the conjugate gradient method was introduced and analyzed. For direct methods, we developed an extension of domain decomposition to handle singular cases encountered in fluid mechanics. In addition, the nested dissection method was adapted to cover the fluid flow case, and its stability was demonstrated. These results are in the following reports:

Deflated conjugate gradients with applications to boundary value problems. SIAM Jnl. Num. An. (1987)

On substructuring algorithms and solution techniques for the numerical approximation of partial differential equations. (with M.D. Gunzburger) App. Num. Math. 2 (1986)

Elimination with noninvertible pivots. (with M.D. Gunzburger) J. Lin. Alg. Applics. 64 (1985)

Applicability of nested dissection to the two and three dimensional Navier-Stokes equations. (with X. Wu) Proc. Intl. Conf. Comp. Eng., (ICCES) Springer-Verlag (1988)

Iterative solution of elliptic boundary value problems on general meshes. (with S. Choudhury) Finite Element Theory and Applics. ed. M.Y. Hussaini, Springer-Verlag (1987)

4. There is considerable interest in flow problems in connection with phase transitions. As a preliminary, some effort was spent on an analysis of the Cahn-Hilliard equation which governs the structure and evolution of the phases. A numerical scheme which has a Lyapunov functional was devloped for the time dependent equation. Analysis of this scheme was completed. An implementation was made. The relevant reports are:

Discrete approximations for structured phase transitions in a finite interval. Proc. IMACS Intnl. Conf. Num. Meth. PDE (1987)

Numerical analysis of a continuum model of phase transition. (with Q. Du) (submitted to SIAM Jnl. Num. An.)

Numerical Results on a Cahn-Hilliard Model of Solidification. (with D. French) App. Num. Math. (1989)

5. A new approach to discretization of the flow equations is under development. This approach fills in the gap caused by the inadequacy of low order finite elements. It is defined for triangular and tetrahedral meshes, and is control volume based. It differs in a key aspect from other schemes in that it uses a single velocity component normal to the faces of the triangulation. Preliminary work is presented in:

Triangular discretization for the vorticity-velocity equations. Proc. 7th Intl. Conf. on Fin. Elems. in Flow Problems, Huntsville, Alabama (1989)

Flow discretization by complementary volume techniques. (AIAA paper 89-1978) Proc. 9th AIAA CFD Meeting, Buffalo, New York (1989)

6. Recently, there has been a surge of activity in numerical algorithms for control of partial differential equations. One major application of the methods is the control of flows. Although this topic was not addressed during this grant period, some aspects of boundary control of the wave equation were analyzed, and an improved algorithm was developed. This is reported in

An algorithm for boundary control of the wave equation. (with M.D. Gunzburger) (to appear in Adv. Applied Math. 1989)

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